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STRUCTURE OF CHEMICALLY NONEQUILIBRIUM FLOWS WITH A SUDDEN CHANGE IN THE TEMPERATURE AND THE CATALYTIC PROPERTIES OF THE SURFACE

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UDC 533.6.011.8

The problem of chemically nonequilibrium flows in the neighborhood of a point where there is a sudden change in the temperature or the catalytic properties of the surface of a body is of undoubted interest from both theoretical and practical standpoints. Thus, the authors of [1-4] studied the effect of discontinuity of the catalytic properties of the surface on flow about the body within the framework of laminar boundary layer theory or the theory of hypersonic viscous shock layers. The problem was examined in [5-7] in a formulation which was the same except for the introduction of a hypothetical boundary layer immediately after the point of discontinuity: with the use of simplifying assumptions, the investigators succeeded in obtaining an analytic solution for the flow functions in the neighborhood behind the point of discontinuity of surface catalytic properties.

To describe the upstream propagation of disturbances from the point of discontinuity – such propagation being absent for boundary-value problems of the parabolic type [1-7] – the authors of [8-10] considered longitudinal diffusion in a certain region of the point; the substantiation for such a flow model for a removable discontinuity was presented in [11], where investigators made use of the method of combinable asymptotic expansions [12]. This method has already been used to solve many problems involving singular perturbations in fluid mechanics (see [13, 14] and their bibliographies, for example).

When analyzing the neighborhood of a point of discontinuity of surface catalytic properties, it is necessary to consider that in the transition from a noncatalytic surface to a surface which is ideally catalytic (for example), the density of the gas near the surface of the body is increased by a characteristic amount, i.e., the streamlines are shifted toward the surface of the body and the flow moves past a hypothetical depression. The main assumption of Prandtl's classical boundary-layer theory – that the longitudinal gradients of the flow functions are small compared to the transverse gradients – may be invalidated for such flows, and it becomes necessary to use the complete Navier–Stokes equations. A systematic analysis of the flow regimes around small two-dimensional irregularities on the surface of a body was performed in [15]. A solution to the problem of surface temperature

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Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 3, pp. 30-41, May-June, 1990. Original article submitted June 9, 1987; revision submitted November 17, 1988.

0021-8944/90/3103-0367$12.50

discontinuity — which also reduces to the analysis of flow past a hypothetical irregularity — was obtained in [16]. However, the results presented in this study pertain to a very small range of cases.

In the current investigation, we study the neighborhood of a point of discontinuity of surface catalytic properties and temperature in the laminar, supersonic, chemically nonequilibrium flow of a binary mixture about a cold plate at Reynolds numbers approaching infinity. It is found that chemical reactions can take place only on the catalytic surface in the neighborhood of the point of discontinuity. We determine the main similarity parameters, present distributions of induced pressure perturbations, shear stress, the normal gradients of enthalpy, and the mass concentration of atoms on the plate, and derive asymptotic laws for the changes in these characteristics.

1. We will examine a uniform, supersonic, chemically nonequilibrium flow of a viscous gas about a flat semi-infinite plate at large but subcritical Reynolds numbers Re_0. It is assumed that the gas is a binary mixture of atoms and diatomic molecules consisting of these atoms. It is further assumed that the temperature of the surface is no higher than the level at which the molecules begin to dissociate at the local pressure. We will study the effect of a sudden change in the temperature and the catalytic properties of the surface of the plate at a certain distance ℓ from its leading edge on the flow past the plate and the plate's heating. We construct a solution to the Navier–Stokes equation together with the mass conservation equation of the atoms at $Re_0 = \rho_0 u_0 \ell/\mu_0 = \varepsilon^{-2} \to \infty$ (the subscript 0 pertains to the flow functions in the uniform incoming flow. We will henceforth use only dimensionless variables. To do this, we refer all of the linear dimensions to the length ℓ, the pressure and shear stress to double the dynamic head in the incoming flow $\rho_0 u_0^2$, the enthalpy to u_0^2, the temperature to mu_0^2/R (m is the molecular weight of the molecular component of the gas and R is the universal gas constant), the heat flux to $\rho_0 u_0^3$, the catalytic coefficient of the surface to u_0, the heat capacities to R/m, and the remaining flow functions to their values in the incoming flow.

Let $\Delta T \sim T \sim O(1)$ and $\Delta c \sim c \sim O(1)$, i.e., the temperature T and mass concentration of atoms c on the surface change within their respective orders of magnitude. Following the method of combinable asymptotic expansions [12], first we examine a region with the characteristic longitudinal and transverse dimensions $x \sim y \sim O(1)$. It is well known that at $\varepsilon \to 0$ the flow in this region is described by equations without dissipative and diffusional terms, while in the case of a plate their solution will be the undisturbed incoming flow. To satisfy the boundary conditions on the surface of the plate, we also introduce a region with the dimensions $x \sim O(1)$ and $y \sim O(\varepsilon)$ — the Prandtl boundary layer. Solutions in this flow region with a discontinuity in the catalytic properties on the surface were obtained in [1-4]. However, these solutions do not describe the small neighborhood of the point of discontinuity.

The sudden change in the temperature and catalytic properties of the surface of the plate may cause a change in the density of the gas $\Delta\rho \sim \rho \sim O(1)$ in the boundary layer. Assuming that the Prandtl and Schmidt numbers $Pr \sim Sc \sim O(1)$ in this case, we further suppose that, in the most general case, the thickness of such a boundary layer is the same as the thicknesses of the viscous, heat-conducting, and diffusion layers in terms of order of magnitude. Then using the estimate for longitudinal velocity $u \sim O(y/\varepsilon)$ (in the boundary layer, the flow functions change in proportion to the distance from the surface) and equating the orders of magnitude of the convective and dissipative terms of the conservation equation for longitudinal momentum, we find an estimate for the thickness of the disturbed boundary layer Δy in relation to its length $\Delta x \lesssim 1$:

$$\Delta y \sim O(\varepsilon \Delta x^{1/3}) \leqslant \varepsilon. \tag{1.1}$$

The change in the thickness of the boundary layer (1.1) leads to a proportional displacement of its external boundary [13] and, through the interaction with the uniform, supersonic incoming flow, initiates the pressure disturbance $\Delta p \sim O(\Delta y/\Delta x)$. Assuming in the general case that the pressure disturbance results in nonlinear perturbations of velocity u in the boundary layer (1.1) — $u \sim \Delta u \sim \Delta p^{1/2}$ — we find that Δp is in agreement with Eq. (1.1) only at

$$\Delta x \sim O(\varepsilon^{3/4}),\ \Delta y \sim O(\varepsilon^{5/4}),\ \Delta p \sim O(\varepsilon^{1/2}). \tag{1.2}$$

These estimates determine the extent of the disturbed region of the point of discontinuity of surface catalytic properties and temperature. Laminar boundary-layer theory and the theory of hypersonic viscous shock layers are now no longer valid for this region, and it is necessary to take into account the induced pressure perturbation. It should also be noted that perturbed boundary layer (1.1) is located on both sides of the point of discontinuity of the boundary conditions, not just downstream – as was assumed in [5-7].

If the flow about the plate is a chemically equilibrium flow, then when its surface temperature is below the value at which the dissociation reaction proceeds at the local pressure, the atomic component will be absent from boundary layer (1.1) and $c \simeq \partial c/\partial y \simeq 0$; in this case, only chemically nonreacting flows can be studied. In the case of nonequilibrium flow, all of the terms in the conservation equations for mass concentration (including the term that describes the rate of formation of an individual component) should be of the same order of magnitude $O(1)$ [17]. Estimates (1.2) show that large gradients of the flow functions are initiated in the investigated region of the point of discontinuity (for example, $\partial p/\partial x \sim O(\varepsilon^{-1/4}) \gg 1$). Thus, the term which describes the rate of formation of an individual component of the gas mixture is unimportant in this case and the flow can be regarded as chemically "frozen," with recombination reactions occurring on the surface of the plate [18]. Since $\Delta p \sim O(\varepsilon^{1/2})$ is an order of magnitude less than the perturbation of temperature or the mass concentration of atoms $T \sim \Delta T \sim c \sim \Delta c \sim O(1)$ in the investigated region, then the effect of barodiffusion is asymptotically small. The term which describes thermodiffusion is of the same order as the convective terms of the conservation equation for the mass concentration of atoms. However, as is customary, we will also ignore it in the present case due to its smallness [17].

2. In accordance with the method of studying interacting flows that was proposed in [13], it is necessary to first examine the perturbed region 1 of a uniform, supersonic incoming flow with the characteristic dimensions $\delta \ll x \sim y \sim O(\varepsilon^{3/4}) \ll 1$ [$\delta \sim O(\varepsilon)$ is the thickness of the boundary layer]. The following independent variables and asymptotic expansions of the flow functions are valid within this region

$$\begin{gathered} x = \varepsilon^{3/4} x_1,\ y = \varepsilon^{3/4} y_1, \\ u = 1 + \varepsilon^{1/2} u_1 + \ldots,\ v = \varepsilon^{1/2} v_1 + \ldots,\ \rho = 1 + \varepsilon^{1/2} \rho_1 + \ldots, \\ p = p_0 + \varepsilon^{1/2} p_1 + \ldots,\ h = h_0 + \varepsilon^{1/2} h_1 + \ldots \end{gathered} \tag{2.1}$$

Substitution of expansions (2.1) into the Navier–Stokes equation and passage to the limit $\varepsilon \to 0$ show that, in a first approximation, the flow in region 1 is described by the linearized Euler equations

$$\frac{\partial \rho_1}{\partial x_1} + \frac{\partial u_1}{\partial x_1} + \frac{\partial v_1}{\partial y_1} = 0,\ \frac{\partial u_1}{\partial x_1} + \frac{\partial p_1}{\partial x_1} = 0,\ \frac{\partial v_1}{\partial x_1} + \frac{\partial p_1}{\partial y_1} = 0,\ \frac{\partial h_1}{\partial x_1} - \frac{\partial p_1}{\partial x_1} = 0. \tag{2.2}$$

System (2.2) is augmented by the formula for determining the Mach number for the "frozen" sonic velocity and the relation for the enthalpy increment in the case of a mixture with a "frozen" chemical composition

$$\mathrm{M}^2 = \frac{1 - \partial h/\partial p}{\partial h/\partial \rho},\ h_1 = \frac{\partial h}{\partial p} p_1 + \frac{\partial h}{\partial \rho} \rho_1. \tag{2.3}$$

Equations (2.2) and (2.3) are readily reduced to a wave equation whose solution (d'Alembert's solution for equations of the hyperbolic type) is well known, and at $y_1 \to 0$

$$\sqrt{\mathrm{M}^2 - 1}\, p_1(x_1, 0) = v_1(x_1, 0). \tag{2.4}$$

Since the flow is chemically "frozen" in the investigated region of the point of discontinuity of the surface properties, then the mass concentrations of the components of the mixture remain constant along the streamlines.

Now we will examine perturbed region 2 of the main part of the boundary layer on a plate with the characteristic dimensions $\delta \ll x \sim O(\varepsilon^{3/4}) \ll 1$, $y \sim O(\varepsilon) \sim \delta$. In this region, we introduce new independent variables and new asymptotic expansions of the flow functions

$$\begin{gathered} x = \varepsilon^{3/4} x_2,\ y = \varepsilon y_2, \\ u = u_{20}(y_2) + \varepsilon^{1/4} u_{21} + \ldots,\ v = \varepsilon^{1/2} v_{21} + \ldots,\ \rho = \rho_{20}(y_2) + \varepsilon^{1/4} \rho_{21} + \ldots, \\ p = p_0 + \varepsilon^{1/2} p_2 + \ldots,\ T = T_{20}(y_2) + \varepsilon^{1/4} T_{21} + \ldots, \\ c = c_{20}(y_2) + \varepsilon^{1/4} c_{21} + \ldots, \end{gathered} \tag{2.5}$$

where $u_{20}(y_2)$, $\rho_{20}(y_2)$, $T_{20}(y_2)$, and $c_{20}(y_2)$ are the profiles of longitudinal velocity, density, temperature, and the mass concentration of atoms in the undisturbed boundary layer on the plate ahead of the point of discontinuity of the boundary conditions. Substitution of expansions (2.5) into the Navier–Stokes equations and the mass conservation equation (see [17], for example) and passage to the limit $\varepsilon \to 0$ show that, in the first approximation, the flow in region 2 is described by the system of equations

$$\rho_{20}\frac{\partial u_{21}}{\partial x_2}+u_{20}\frac{\partial \rho_{21}}{\partial x_2}+\rho_{20}\frac{\partial v_{21}}{\partial y_2}+v_{21}\frac{d\rho_{20}}{dy_2}=0,\quad u_{20}\frac{\partial u_{21}}{\partial x_2}+v_{21}\frac{du_{20}}{dy_2}=0,$$
$$\frac{\partial p_2}{\partial y_2}=0,\quad u_{20}\frac{\partial T_{21}}{\partial x_2}+v_{21}\frac{dT_{20}}{dy_2}=0,\quad u_{20}\frac{\partial c_{21}}{\partial x_2}+v_{21}\frac{dc_{20}}{dy_2}=0,$$

which allows partial integration

$$u_{21}=Ddu_{20}/dy_2,\ v_{21}=-u_{20}dD/dx_2,\ \rho_{21}=Dd\rho_{20}/dy_2,$$
$$T_{21}=DdT_{20}/dy_2,\ c_{21}=Ddc_{20}/dy_2,\ p_2=p_2(x_2) \tag{2.6}$$

[$D = D(x_2)$ is an arbitrary function].

In the viscous, heat-conducting, and diffusion boundary layer 3, with the characteristic dimensions $\delta \ll x \sim O(\varepsilon^{3/4}) \ll 1$ and $y \sim O(\varepsilon^{5/4}) \ll \delta$, we introduce the following independent variables and asymptotic expansions of the flow functions

$$x=\varepsilon^{3/4}x_3,\ y=\varepsilon^{5/4}y_3,\ u=\varepsilon^{1/4}u_3+\ldots,\ v=\varepsilon^{3/4}v_3+\ldots,$$
$$\rho=\rho_3+\ldots,\ p=p_0+\varepsilon^{1/2}p_3+\ldots,\ T=T_{30}+\varepsilon^{1/4}T_{31}+\ldots, \tag{2.7}$$
$$c=c_{30}+\varepsilon^{1/4}c_{31}+\ldots$$

Substitution of expansions (2.7) into the Navier–Stokes equations and mass conservation equation and passage to the limit $\varepsilon \to 0$ show that, in the first approximation, flow in boundary layer 3 is described by the equations of a compressible boundary layer

$$\frac{\partial(\rho_3 u_3)}{\partial x_3}+\frac{\partial(\rho_3 v_3)}{\partial y_3}=0,\quad \rho_3\left(u_3\frac{\partial u_3}{\partial x_3}+v_3\frac{\partial u_3}{\partial y_3}\right)+\frac{dp_3}{dx_3}=\frac{\partial}{\partial y_3}\left(\mu\frac{\partial u_3}{\partial y_3}\right),$$
$$\frac{\partial p_3}{\partial y_3}=0,\quad \rho_3\left(u_3\frac{\partial c_{30}}{\partial x_3}+v_3\frac{\partial c_{30}}{\partial y_3}\right)=\frac{\partial}{\partial y_3}\left(\frac{\mu}{\mathrm{Sc}}\frac{\partial c_{30}}{\partial y_3}\right),\quad p_0=\rho_3T_{30}(1+c_{30}), \tag{2.8}$$
$$\rho_3c_p\left(u_3\frac{\partial T_{30}}{\partial x_3}+v_3\frac{\partial T_{30}}{\partial y_3}\right)=\frac{\partial}{\partial y_3}\left(\frac{\mu c_p}{\mathrm{Pr}}\frac{\partial T_{30}}{\partial y_3}\right)+\frac{\mu}{\mathrm{Sc}}\left(c_{p_1}-c_{p_2}\right)\frac{\partial T_{30}}{\partial y_3}\frac{\partial c_{30}}{\partial y_3}$$

(c_{p_1}, c_{p_2}, c_p are the isobaric heat capacities of the atomic and molecular components and their mixture, respectively.) It is evident from Eq. (2.7) that the terms which describe barodiffusion $\sim\varepsilon^2\partial^2p/\partial x^2 \sim \varepsilon \ll \rho u\partial c/\partial x \sim \varepsilon^{-1/2}$ and lengthwise diffusion $\sim\varepsilon^2\partial^2c/\partial x^2 \sim \varepsilon^{1/2} \ll \varepsilon^2\partial^2c/\partial y^2 \sim \varepsilon^{-1/2}$ should not be in Eq. (2.8).

The external boundary conditions are obtained by combining the asymptotic expansions of flow functions (2.5) and (2.7) in regions 2 and 3 [with the use of Eqs. (2.6)]

$$u_3\to A(y_3+D),\ T_{30}\to T_{20}(0),\ c_{30}\to c_{20}(0)\ (y_3\to\infty) \tag{2.9}$$

($A = du_{20}/dy_2$ for $y_2 = 0$), while the condition for determination of the pressure is obtained by combining the asymptotic expansions for pressure (2.1), (2.5), and (2.7) in regions 1, 2, and 3 [with the use of Eqs. (2.4) and (2.6)]

$$p_3(x_3)=p_2(x_2)=p_1(x_1,0)=\frac{v_1(x_1,0)}{\sqrt{M^2-1}}=\frac{v_{21}(x_2,\infty)}{\sqrt{M^2-1}}=-\frac{dD/dx_2}{\sqrt{M^2-1}}. \tag{2.10}$$

The initial boundary conditions are derived from combining the above with the wall portion of the undisturbed boundary layer on the plate:

$$u_3\to Ay_3,\ T_{30}\to T_{20}(0),\ c_{30}\to c_{20}(0),\ p_3,\ D\to 0\ (x_3\to-\infty). \tag{2.11}$$

Conditions of adhesion and impermeability should be satisfied on the plate surface for the components of velocity

$$u_3=v_3=0\ (y_3=0). \tag{2.12}$$

If the origin of the coordinates of region 3 is placed at the point where the temperature and catalytic properties of the plate surface change suddenly, then the conditions for the temperature on the surface are written in the form

$$T_{30} = T_{20}(0)\ (x_3 < 0),\ T_{30} = (1+\alpha)T_{20}(0)\ (x_3 \geqslant 0,\ y_3 = 0). \tag{2.13}$$

Here, the parameter $\alpha > -1$ characterizes the size of the pressure change.

The condition for the mass concentration of atoms on the catalytic surface is as follows [17] for the investigated case of flow about a plate

$$\partial c/\partial y = \mathrm{Sc}\, k\rho c/\varepsilon^2\mu, \tag{2.14}$$

where the coefficient characterizing catalytic activity $k = \varepsilon^\beta K$, $K \sim O(1)$. Since the flow in the investigated region is chemically "frozen," it makes sense to deal only with those cases in which the plate surface ahead of the point of boundary-condition discontinuity is not ideally catalytic (i.e., $c_{20}(0) > 0$ and $\beta = 1$). Otherwise, the atomic component will be absent from region 3. Then, in the variables for region 3, boundary condition (2.14) will have the following form at $x_3 < 0$

$$\partial c_{30}/\partial y_3 = 0\ (x_3 < 0,\ y_3 = 0). \tag{2.15}$$

It immediately follows from Eqs. (2.8) and boundary conditions (2.9), (2.11), (2.13), and (2.15) that

$$T_{30}(x_3, y_3) \equiv T_{20}(0),\ c_{30}(x_3, y_3) \equiv c_{20}(0)\ (x_3 < 0). \tag{2.16}$$

It is obvious that a nontrivial solution for the mass concentration of the atoms at $x_3 > 0$ can be found only with values of the parameter $\beta \leq 3/4$. Then, in the variables for region 3, boundary condition (2.14) is represented in the form

$$\partial c_{30}/\partial y_3 = \mathrm{Sc}\, K\rho_3 c_{30}/\mu \quad \text{or} \quad c_{30} = 0\ \ (x_3 \geqslant 0,\ \ y_3 = 0). \tag{2.17}$$

At $x_3 < 0$ and with satisfaction of Eq. (2.16), it is best to examine the following terms of expansions (2.7) for the temperature and mass concentration of atoms

$$\rho_3\left(u_3\frac{\partial c_{31}}{\partial x_3} + v_3\frac{\partial c_{31}}{\partial y_3}\right) = \frac{\partial}{\partial y_3}\left(\frac{\mu}{\mathrm{Sc}}\frac{\partial c_{31}}{\partial y_3}\right),\ \rho_3 c_p\left(u_3\frac{\partial T_{31}}{\partial x_3} + v_3\frac{\partial T_{31}}{\partial y_3}\right) = \frac{\partial}{\partial y_3}\left(\frac{\mu c_p}{\mathrm{Pr}}\frac{\partial T_{31}}{\partial y_3}\right). \tag{2.18}$$

It is evident that barodiffusion and lengthwise diffusion are unimportant in this approximation as well.

The external boundary conditions for the functions c_{31} and T_{31} are derived by combining the asymptotic expansions of the flow functions (2.5) and (2.7) in regions 2 and 3 [with the use of Eqs. (2.6)]:

$$T_{31} \to B(y_3 + D),\ c_{31} \to C(y_3 + D)\ (y_3 \to \infty), \tag{2.19}$$

where $B = dT_{20}/dy_2$ and $C = dc_{20}/dy_2$ at $y_2 = 0$. We find the initial conditions by combining the above with the wall portion of the undisturbed boundary layer on the plate:

$$T_{31} \to By_3,\ c_{31} \to Cy_3\ (x_3 \to -\infty). \tag{2.20}$$

On the plate surface, the functions c_{31} and T_{31} must satisfy the conditions

$$\partial c_{31}/\partial y_3 = C,\ T_{31} = 0\ (x_3 < 0,\ y_3 = 0). \tag{2.21}$$

In the variables (2.7), the shear stress τ and heat flux q are expressed by the formulas

$$\tau = \varepsilon\mu\partial u_3/\partial y_3 + \ldots,\ -q = \varepsilon^{3/4}q_1 + \varepsilon q_2 + \ldots,$$

$$q_1 = \frac{\mu}{\mathrm{Pr}}\left[c_p\frac{\partial T_{30}}{\partial y_3} + \mathrm{Le}\frac{\partial c_{30}}{\partial y_3}\left(\int\limits_0^{T_{30}}(c_{p_1} - c_{p_2})\,dT + h^0\right)\right], \tag{2.22}$$

$$q_2 = \frac{\mu}{\mathrm{Pr}}\left[c_p\frac{\partial T_{31}}{\partial y_3} + \mathrm{Le}\frac{\partial c_{31}}{\partial y_3}\left(\int\limits_0^{T_{30}}(c_{p_1} - c_{p_2})\,dT + h^0\right)\right].$$

Here, Le is the Lewis–Semenov number: h^0 is the dimensionless heat of formation of the atoms. Equations (2.22) show that the distribution of the heat flux q at $x_3 < 0$ is described by the second term [see Eq. (2.16)], which is of the same order as the heat flux in the undisturbed boundary layer on the plate. The sudden change in the temperature and catalytic properties of the surface lead to a change in order of magnitude of the heat flux, and at $x_3 \geq 0$ its distribution will be determined mainly by the first term.

To reduce boundary-value problem (2.8)-(2.13), (2.15), (2.17)-(2.21) to a form convenient for numerical integration, we make the substitution of variables

$$\begin{gathered} x = \frac{x_3}{x^*},\ \eta = \frac{1}{\eta^*}\int_0^{y_3} \rho_3\, dy_3,\ \psi = \frac{\psi_3}{\psi^*},\ \frac{\partial\psi_3}{\partial y_3} = \rho_3 u_3,\ \frac{\partial\psi_3}{\partial x_3} = -\rho_3 v_3, \\ p = \frac{p_3}{p^*},\ c_1 = c_{30},\ T_1 = \frac{T_{30}}{T_{20}(0)\,[1 + c_{20}(0)]},\ c_2 = \frac{c_{31}\rho^*}{C\eta^*},\ T_2 = \frac{T_{31}\rho^*}{B\eta^*}, \\ d = \frac{D\rho^*}{\eta^*},\ \rho^* = \frac{p_0}{T_{20}(0)\,[1 + c_{20}(0)]},\ x^* = \rho^{*-1/4}(\mathrm{M}^2 - 1)^{-3/8}A^{-5/4}, \\ \eta^* = \rho^{*1/4}(\mathrm{M}^2 - 1)^{-1/8}A^{-3/4},\ \psi^* = \rho^{*-1/2}(\mathrm{M}^2 - 1)^{-1/4}A^{-1/2},\ p^* = A^{1/2}\rho^{*-1/2}(\mathrm{M}^2 - 1)^{-1/4} \end{gathered} \tag{2.23}$$

and we adopt the simplifying assumptions [17]

$$\rho_3\mu = 1,\ \mathrm{Pr},\ \mathrm{Sc},\ c_p = \mathrm{const},\ \int_0^{T_{30}} \left(c_{p_1} - c_{p_2}\right) dT \ll h^0. \tag{2.24}$$

Then, with assumptions (2.24), the boundary-value problem is written in the following standard form in variables (2.23)

$$\begin{gathered} \psi''' = -d^{\cdot}\cdot(1 + c_1)T_1 + \psi'\psi'^{\cdot} - \psi^{\cdot}\psi'', \\ c_i''/\mathrm{Sc} = \psi' c_i^{\cdot} - \psi^{\cdot} c_i',\ T_i''/\mathrm{Pr} = \psi' T_i^{\cdot} - \psi^{\cdot} T_i',\ i = 1, 2, \\ \psi(x, 0) = 0,\ \psi'(x, 0) = 0,\ c_2'(x, 0) = 1,\ T_2(x, 0) = 0, \\ c_1'(x, 0) = F\frac{c_1(x, 0)}{1 + c_1(x, 0)},\ T_1(x, 0) = \frac{1 + \alpha}{1 + c_{20}(0)} \quad (x \geqslant 0), \\ \psi'(x, \infty) \to \int_0^\infty T_1(1 + c_1)\, d\eta + d,\ c_1(x, \infty) \to c_{20}(0),\ T_1(x, \infty) \to \frac{1}{1 + c_{20}(0)}, \\ c_2'(x, \infty) \to 1,\ T_2'(x, \infty) \to 1,\ \psi(-\infty, \eta) \to \eta^2/2,\ c_2(-\infty, \eta) \to \eta, \\ T_2(-\infty, \eta) \to \eta,\ d(-\infty) \to 0,\ c_1(x, \eta) = c_{20}(0), \\ T_1(x, \eta) = 1/(1 + c_{20}(0))\ (x < 0), \quad F = \frac{Sc\, K p_0^{5/4}}{(\mathrm{M}^2 - 1)^{1/8} A^{3/4} T_{20}^{5/4}(0)\,(1 + \alpha)\,[1 + c_{20}(0)]^{1/4}}. \end{gathered} \tag{2.25}$$

Here, $(\ )^{\cdot} = \partial/\partial x$; $(\ )' = \partial/\partial\eta$; F is the local Damköhler number. Meanwhile, the formulas for the shear stress and heat flux (2.22) reduce to the form

$$\begin{gathered} \tau = \psi'' + \ldots,\ q_1 = T_1' + \mathrm{Le}\, E_1 c_1',\ q_2 = T_2' + \mathrm{Le}\, E_2 c_2', \\ E_1 = h^0/c_p T_{20}(0)[1 + c_{20}(0)],\ E_2 = Ch^0/Bc_p, \end{gathered} \tag{2.26}$$

where τ, q_1, and q_2 are additionally referred to the quantities $\varepsilon A/\rho^*$, $c_p T_{20}(0)[1 + c_{20}(0)]/\mathrm{Pr}\,\eta^*$ and $c_p B/\mathrm{Pr}\,\rho^*$, respectively. The similarity parameters E_1 and E_2 characterize the ratio of the heat-flux components due to heat conduction and energy transport by the diffusing constituents of the gas mixture.

Intensification of the catalytic activity of the plate surface at $x \geq 0$ should obviously lead to a reduction in the mass concentration of atoms near this surface, i.e., $c_1(x, 0) \to 0$ at $x \to \infty$. Then the use of the variables

$$n = \eta/(3x)^{1/3},\ \psi = (3x)^{2/3}(\varphi + J_2 + \gamma_1 n),\ d = (3x)^{1/3}\gamma_1 \tag{2.27}$$

makes it possible to reduce boundary-value problem (2.25) to a similarity problem (with $x \to \infty$)

$$\begin{gathered} \frac{d^3\varphi}{dn^3} + 2(\varphi + \gamma_1 n + J_2)\frac{d^2\varphi}{dn^2} - \left(\frac{d\varphi}{dn} + 2\gamma_1 + 2J\right)\frac{d\varphi}{dn} + 2T_1(1 + c_1)\varphi = \\ = (\gamma_1 + J)^2 - \frac{d}{dn}[T_1(1 + c_1)] - 2T_1(1 + c_1)(\gamma_1 n + J_2), \end{gathered}$$

$$\frac{1}{\mathrm{Sc}}\frac{d^2c_1}{dn^2}+2(\varphi+\gamma_1 n+J_2)\frac{dc_1}{dn}=0,\ \frac{1}{\mathrm{Pr}}\frac{d^2T_1}{dn^2}+2(\varphi+\gamma_1 n+J_2)\frac{dT_1}{dn}=0, \tag{2.28}$$

$$\varphi=0,\ d\varphi/dn=-\gamma_1,\ c_1=0,\ T_1=(1+\alpha)/[1+c_{20}(0)]\ (n=0),$$

$$d\varphi/dn=0,\ d^2\varphi/dn^2=0,\ c_1=c_{20}(0),\ T_1=1/[1+c_{20}(0)]\ (n\to\infty),$$

$$J=\int_0^n T_1(1+c_1)\,dn,\quad J_2=\int\int_0^n T_1(1+c_1)\,dn\,dn.$$

One feature of boundary-value problem (2.28) is the absence of the term with the longitudinal pressure gradient. There is also another unknown parameter γ_1, which is found from the solution of this problem. The form of the variables (2.27) shows that at $x \to \infty$ the shear stress τ [see Eq. (2.26)] approaches its value in the undisturbed boundary layer on the plate ahead of the neighborhood of the point of boundary-condition discontinuity. Meanwhile, the heat flux q_1 decreases in accordance with the law

$$\tau=\psi''=\frac{d^2\varphi}{dn^2}+T_1(1+c_1),\ q_1=\frac{1}{(3x)^{1/3}}\left(\frac{dT_1}{dn}+\mathrm{Le}\,E_1\frac{dc_1}{dn}\right). \tag{2.29}$$

The asymptote of solutions of boundary-value problems of the type (2.25) at $x \to -\infty$ was studied in [13]. Equations (2.27) are functionally similar to the expansions constructed in [6, 7] for the internal boundary layer immediately beyond the point of discontinuity of the properties of the plate surface. However, the authors of these studies did not consider the effect of the change in the properties of the surface on the perturbations of pressure, shear stress, and heat flux, i.e., they only examined cases for a removable discontinuity.

3. To analyze the solution of boundary-value problem (2.25) at $x \to 0$, it is necessary to additionally study region 4. This region, located around the sudden change in the temperature and catalytic properties of the plate surface, has a size characterized as $\varepsilon^{3/2} < x < \varepsilon^{3/4}$. If the change in the characteristics of the plate surface do not result in separation of the boundary layer, then the same relations that were valid for region 3 are valid for the nonlinear viscous, heat-conducting, and diffusion region 4. Region 4 is located entirely within region 3, and the external flow for region 4 is the flow in the wall part of region 3 at $x_3 \to -0$:

$$u_3=A_w y_3,\ v_3=0,\ p_3=p_w,\ \rho_3=\frac{p_0}{T_{20}(0)\,[1+c_{20}(0)]},\ T_{30}=T_{20}(0),\ c_{30}=c_{20}(0). \tag{3.1}$$

Here, $A_w = (\partial u_3/\partial y_3)_w$; the subscript w denotes quantities at the surface of the plate with $x_3 \to -0$. The independent variables and asymptotic expansions of the flow functions (2.7) are valid in region 4

$$x=\Delta x x_4,\ y=\varepsilon\Delta x^{1/3}y_4,$$
$$u=\Delta x^{1/3}u_4+\ldots,\ v=(\varepsilon/\Delta x^{1/3})v_4+\ldots,\ \rho=\rho_4+\ldots,\ c=c_{40}+\ldots, \tag{3.2}$$
$$p=p_0+\varepsilon^{1/2}p_w+\Delta x^{2/3}p_4+\ldots,\ T=T_{40}+\ldots$$

Substitution of Eq. (3.2) into the Navier–Stokes equations and the mass conservation equation and passage to the limit $\varepsilon \to 0$, $\varepsilon^{3/2} < \Delta x < \varepsilon^{3/4}$ shows that, in the first approximation, the flow in region 4 is described by equations of the form (2.8) (the subscript 3 should be replaced by 4). The external and initial boundary conditions are obtained by combining expansions (3.2) with the wall part of region 3 at $x_3 \to -0$ (3.1) and have the form (2.9), (2.11) (the subscript 3 is replaced by 4, $D \equiv 0$, and we use A_w instead of A).

In order to obtain a nontrivial solution to the mass conservation equation at $x_4 > 0$, it is necessary that $\varepsilon^\beta \gtrsim \varepsilon/\Delta x^{1/3}$. For lower values of the catalytic activity coefficient, there will not be sufficient time for the mass concentration of atoms to change its order of magnitude over the interval $\varepsilon^{3/2} < \Delta x < \varepsilon^{3/4}$. Then the boundary conditions on the plate surface take the form (2.12), (2.13), (2.15), (2.17) (with the corresponding replacement of 3 by 4).

There will be no interaction with the uniform supersonic incoming flow in this case, since otherwise $\Delta p \sim \Delta y/\Delta x \sim \varepsilon/\Delta x^{2/3} > \Delta x^{2/3}$ would be induced at $\Delta x < \varepsilon^{3/4}$. Thus, a compensative flow regime is realized in region 4 [15]. Here, the change in the thickness of region 4 is, in a first approximation, compensated for by the change in the thickness of the wall part of the flow in region 3 immediately ahead of the point where the properties of the plate surface change. It was found in [15] that, in this case, the induced pressure disturbance is determined from the relation

$$\rho_{20}(0)A_w v_4 + dp_4/dx_4 \to 0 \ (y_4 \to \infty). \tag{3.3}$$

A characteristic feature of boundary-value problem (2.8), (2.9), (2.11)-(2.13), (2.15), (2.17) (after the substitution of 4 for 3, A_w for A, and $D \equiv 0$) and Eq. (3.3) is the latter compensating condition of the interaction for determination of the pressure perturbation. However, the order of Eq. (3.3) is no greater than the order of the derivatives, with respect to the longitudinal coordinate, that enter into this problem, and the condition does not induce perturbations ahead of the point of discontinuity of the boundary conditions [15] (the problem remains parabolic). As a result, initial boundary conditions can be assigned at $x_4 = 0$ and we can study the solution at $x_4 > 0$. Equations (2.22) for the shear stress τ and the heat flux q take the following form in variables (3.2)

$$\tau = \varepsilon\mu \frac{\partial u_4}{\partial y_4} + \ldots, -q = \frac{\varepsilon}{\Delta x^{1/3}} \frac{\mu}{\Pr}\left[c_p \frac{\partial T_{40}}{\partial y_4} + \mathrm{Le}\frac{\partial c_{40}}{\partial y_4}\left(\int\limits_0^{T_{40}} (c_{p_1} - c_{p_2})\, dT + h^0\right)\right]. \tag{3.4}$$

Introduction of the new variables

$$x = \frac{x_4}{x^*}, n = \frac{1}{\eta^* (3x)^{1/3}} \int\limits_0^{y_4} \rho_4\, dy_4,\ \ \psi_4 = \psi^* (3x)^{2/3} (\varphi + J_2),$$
$$\frac{\partial \psi_4}{\partial y_4} = \rho_4 u_4, \frac{\partial \psi_4}{\partial x_4} = -\rho_4 v_4,\ \ p_4 = p^* (3x)^{2/3} \gamma_2, c_1 = c_{40}, T_1 = \frac{T_{40}}{T_{20}(0)\,[1 + c_{20}(0)]}, \tag{3.5}$$
$$\eta^* = x^{*1/3} \rho_{20}^{1/3}(0)\, A_w^{-1/3}, \psi^* = x^{*2/3} \rho_{20}^{-1/3}(0)\, A_w^{1/3},\ \ p^* = x^{*2/3} \rho_{20}^{-1/3}(0)\, A_w^{4/3}$$

and the use of simplifying assumptions (2.24) makes it possible to reduce boundary-value problem (2.8), (2.9), (2.11)-(2.13), (2.15), (2.17) [with the corresponding changes for region 4], and (3.3) in partial derivatives to the similarity problem

$$\frac{d^3\varphi}{dn^3} + 2(\varphi + J_2)\frac{d^2\varphi}{dn^2} - \left(\frac{d\varphi}{dn} + 2J\right)\frac{d\varphi}{dn} + 2T_1(1 + c_1)\,\varphi = J^2 - \frac{d}{dn}[T_1(1 + c_1)] - 2T_1(1 + c_1)(J_2 + \gamma_2),$$
$$\frac{1}{\mathrm{Sc}} \frac{d^2 c_1}{dn^2} + 2(\varphi + J_2)\frac{dc_1}{dn} = 0, \frac{1}{\Pr}\frac{d^2 T_1}{dn^2} + 2(\varphi + J_2)\frac{dT_1}{dn} = 0, \tag{3.6}$$
$$\varphi = 0, d\varphi/dn = 0, c_1 = 0, T_1 = (1 + \alpha)/[1 + c_{20}(0)]\ (n = 0),$$
$$\varphi = \gamma_2,\ \ d^2\varphi/dn^2 = 0,\ \ c_1 = c_{20}(0),\ \ T_1 = 1/[1 + c_{20}(0)]\ \ (n \to \infty),$$

where γ_2 is an additional unknown parameter found from the solution of this problem. It is evident that at $\varepsilon^\beta \sim \varepsilon/\Delta x^{1/3}$, $\varepsilon^{3/2} < \Delta x < \varepsilon^{3/4}$ the solution of boundary-value problem (3.6) describes the asymptotic behavior of the solution of the initial nonsimilar boundary-value problem for region 4 when $c_{40}(x_4, 0) \to 0$, i.e., when $x_4 \to \infty$, while at $\varepsilon^\beta > \varepsilon/\Delta x^{1/3}$, $\varepsilon^{3/2} < \Delta x < \varepsilon^{3/4}$, i.e., when $c_{40}(x_4, 0) \equiv 0$ these boundary-value problems are identical. In the variables (3.5), Eqs. (3.4) for τ and q take the form

$$\tau = \frac{d^2\varphi}{dn^2} + T_1(1 + c_1) + \ldots, -q = \frac{1}{(3x)^{1/3}}\left(\frac{dT_1}{dn} + \mathrm{Le}\, E_1 \frac{dc_1}{dn}\right) + \ldots \tag{3.7}$$

Here, τ and q are additionally referred to as $\varepsilon A_w/\rho_{20}(0)$ and $(\varepsilon/\Delta x^{1/3}) c_p\, T_{20}(0)[1 + c_{20}(0)]/\Pr \eta^*$, respectively. Equations (3.7) determine the behavior of the solution of boundary-value problem (2.25) at $x \to +0$. It can be seen that Eq. (3.5) and the expansions constructed in [6, 7] for a small region behind the point of discontinuity of surface catalytic properties are functionally similar. As was noted above, such a flow scheme is valid only for a removable surface-property discontinuity. In the general case, it is necessary to solve self-similar boundary-value problem (3.6), which also determines the initial conditions for the internal layer behind the point of boundary-condition discontinuity.

Boundary-value problem (3.6) describes the sudden change in the flow functions with the transition past the point of discontinuity of the boundary conditions on the plate surface. In order to study a neighborhood of this point even smaller than regions 3 and 4, in accordance with the method of combinable asymptotic expansions [12] it is necessary to examine a region whose characteristic length and thickness are of the same order of magnitude — $\Delta x \sim \Delta y$. Estimates (1.1) show that in this case $\Delta x \sim \Delta y \sim O(\varepsilon^{3/2})$, $u \sim v \sim O(\varepsilon^{1/2})$, $\Delta p \sim O(\varepsilon)$, and the flow is described by the complete system of Navier—Stokes equations and conservation equation for the mass concentration of atoms with variable density. Only in this region does lengthwise diffusion become important, i.e., the term $\sim\partial^2 c/\partial x^2$ appears. However, the condition of adhesion on the plate surface will no longer be satisfied, since, due to the finite perturbation of temperature or the mass concentration of atoms $\Delta T \sim T \sim \Delta c \sim c \sim O(1)$, a slip velocity $U \sim \varepsilon^2 \partial T/\partial x \sim O(\varepsilon^{1/2})$ which is comparable to the gas flow velocity develops. Other phenomena of molecular gas dynamics will also be important [19]. Thus, allowing for lengthwise diffusion — as was done in [8-11] — is justified only for removable discontinuities of plate surface properties.

4. In the numerical integration of boundary-value problem (2.25), interaction of the flows in regions 3 and 1 make it necessary to determine the induced pressure during the computation. Allowance for the interaction gives the problem the property of slight ellipticity [13], and in order to obtain a unique solution it is necessary to assign an additional boundary condition where $x \to \infty$ [it follows from the form of the variables (2.27) that $p \to 0$ at $x \to \infty$]. Also, due to the discontinuity of the internal boundary conditions at $x = 0$ in the theoretical region, for $x > 0$ it is necessary to introduce a subregion around the surface of the plate and develop a special procedure for performing calculations in two regions [20, 21].

Numerical calculations of boundary-value problem (2.25) were performed with the following as the governing similarity parameters:

1) $F = 1/\sqrt{2}$, $\alpha = 0$, $c_{20}(0) = 1$;

2) $F \to \infty$, $\alpha = 0$, $c_{20}(0) = 1$;

3) $F = 0$, $\alpha = 1$, $c_{20}(0) = 1$.

With a constant temperature for the plate surface (variants 1 and 2), a reduction in the concentration of atoms near the catalytic surface causes an increase in density and displacement of the streamlines toward the plate surface, i.e., an effective depression is formed here and low-pressure flow is realized. An increase in the temperature of the surface, conversely, causes a reduction in the density of the gas and displacment of the streamlines upward from the plate (variant 3). In this case, the flow moves around an effective projection and compressional flow is realized.

Lines 1-3 in Figs. 1-4 correspond to the numbers of the variants. The behavior of the solution of boundary-value problem (2.25) at $x \to -\infty$ was studied in [13], while at $x \to -0$ all of the flow functions change in a nonsingular manner and take finite values.

Figure 1 shows the distribution of induced pressure p. The character of the pressure change at $x \to +0$ is determined by Eq. (3.5). Numerical solution of boundary-value problem (3.6) makes it possible to find values of unknown parameter $\gamma_2 = 0.22912$ and -0.46593 for variants 2 and 3, respectively. At $x \to \infty$, the distribution of the pressure perturbation is described by Eq. (2.27) ($p = -d^{\cdot}$), while values of the unknown parameter $\gamma_1 = 0.46094$ and -0.81717 are obtained from numerical solution of boundary-value problem (2.28) for variants 2 and 3.

Figure 2 shows the distributions of shear stress τ along the plate surface. The value of τ increases in front of the point of boundary-condition discontinuity in the case of low-pressure flow (curves 1 and 2), while it decreases in the case of compressional flow (curve 3). Solution of local boundry-vlue problem (3.6) at $x \to +0$ yields a subsonic shear flow with variable density moving past an effective depression (variants 1, 2) or projection (variant 3). Thus, at $x \to +0$, τ decreases sharply in the first case and increases in the second. The distribution of τ is continuous for variant 1, while for variants 2 and 3 the solution of boundary-value problem (3.6) yields a shear stress with $\tau = 0.720$ and 0.964 at $x \to +0$, respectively. These values agree well with the results of integration of boundry-value problem (2.25). At $x \to \infty$, the shear stress approaches its value in the undisturbed boundary layer on the plate in front of the point of boundary-condition discontinuity ($\tau \to 1$).

An analysis of curve 2 shows that the maximum possible sudden increase in the catalytic activity of the surface corresponds to an increase in the shear stress by a factor of about 1.5 at $x \to -0$. A sudden increase in the temperature of the surface by a factor of

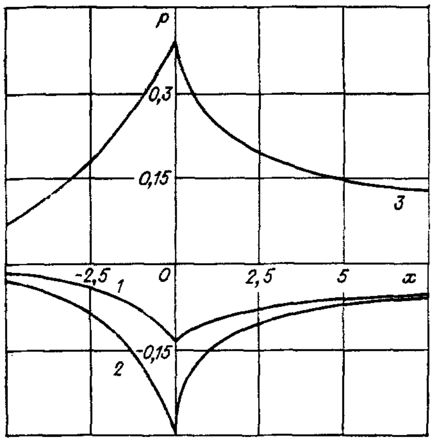

Fig. 1

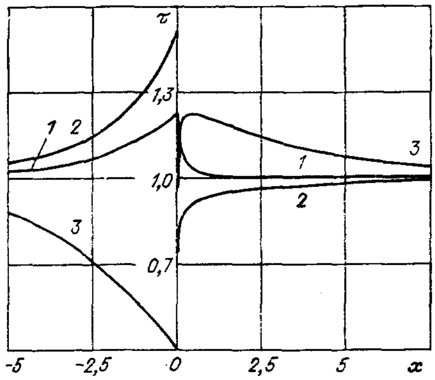

Fig. 2

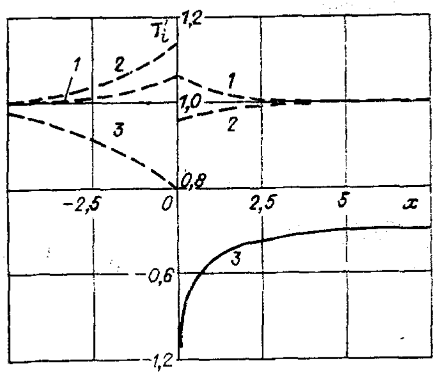

Fig. 3

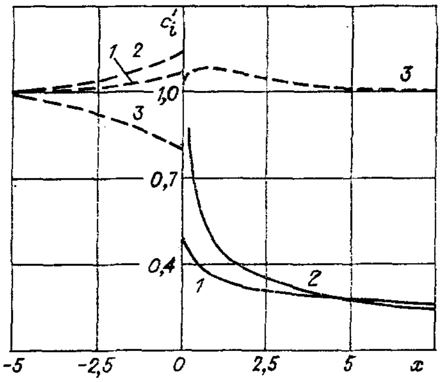

Fig. 4

two lowers τ to 0.4 at $x \to -0$ (curve 3), while an increase in temperature by a factor of about 3.87 leads to the beginning of boundary-layer separation [16].

Figures 3 and 4 show the distributions of the normal gradients of temperature T_i' and the mass concentration of atoms c_i' over the surface of the plate ($i = 1$ for the solid lines, $i = 2$ for the dashed lines). It is evident that a sudden increase in the catalytic activity of the plate surface only slightly alters the normal gradient of temperature T_2' (curves 1 and 2). A sudden doubling of surface temperature lowers T_2' by about 20% at $x \to -0$ (dashed curve 3). At $x > 0$, the highly heated surface is cooled by the incoming flow. Here, the normal temperature gradient changes in order of magnitude and $T_1' < 0$ (solid curve 3). At $x \to \infty$, perturbations from the sudden change in surface properties die out and $T_1' \to 0$, $T_2' \to 1$.

The distributions of c_2' at $x \leq 0$ are similar to the distributions of τ or T_2' for the corresponding variants (dashed lines 1-3 in Fig. 4). A sudden increase in the catalytic activity of the surface at $x > 0$ increases the order of magnitude of the normal gradient of the mass concentration of atoms [solid lines 1 and 2, Eq. (2.22)]. A sudden doubling of the surface temperature only slightly changes the distribution of c_2' (line 3), and at $x \to \infty$, $c_1' \to 0$, $c_2' \to 1$.

The qualitative similarity between the results of our calculations and those presented in [6, 7], for example, follows from the functional similarity of the representations of the solutions. A quantitative comparison of these two sets of results is impossible, however, since different initial conditions and length scales were used in the calculations.

We express thanks to O. G. Fridlender for his helpful consultations.

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